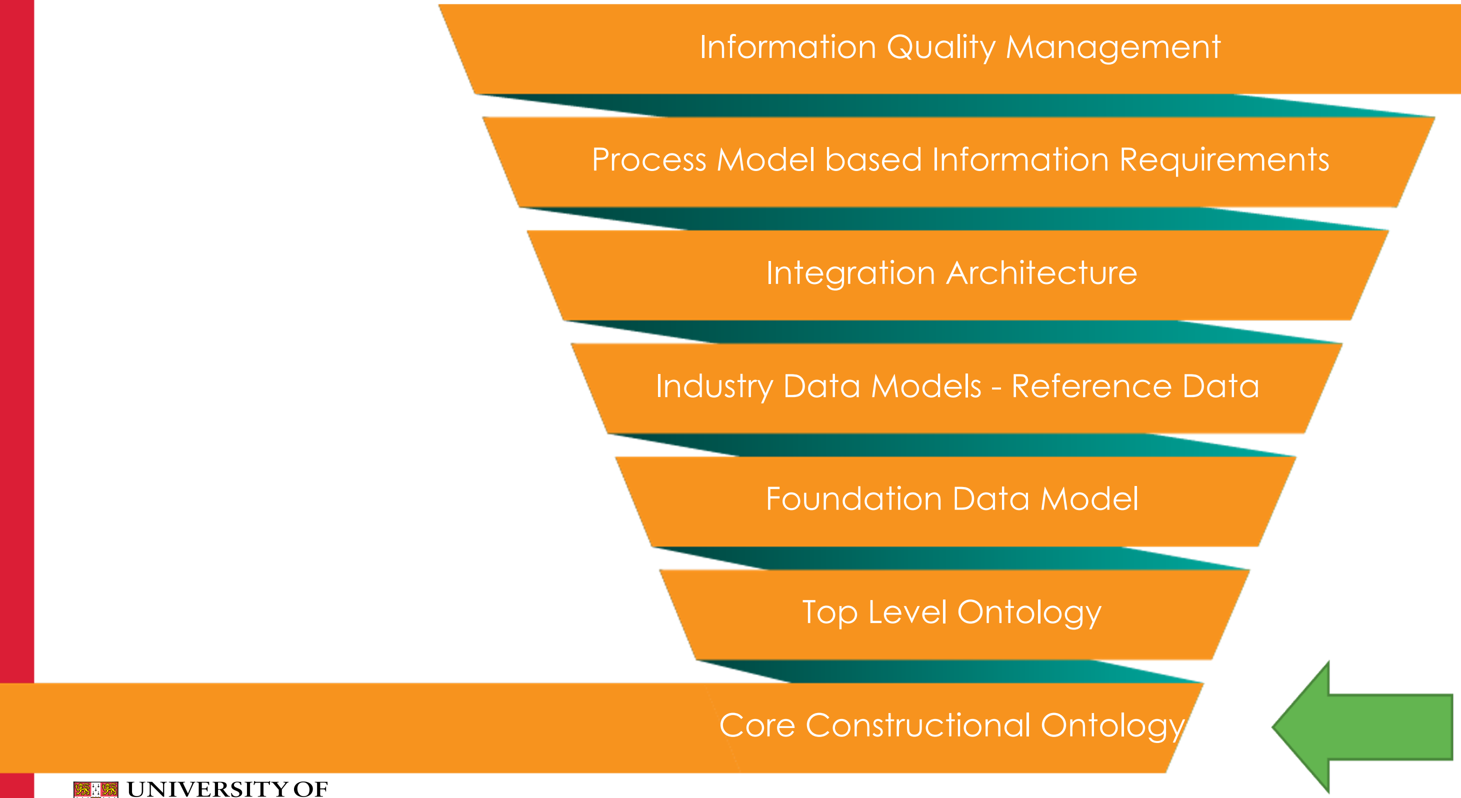
Core Constructional Ontology (CCO): a Constructional Theory of Parts, Sets, and Relations

4-Dimensionalism in Large Scale Data Sharing and Integration

Newton Gateway to Mathematics

The Seven Circles of Information Management



Team effort

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Plan

1. Constructional ontology: background
2. Our approach

    2.1 Key features of the approach

    2.2 Overview of the formalisation

Disclaimer

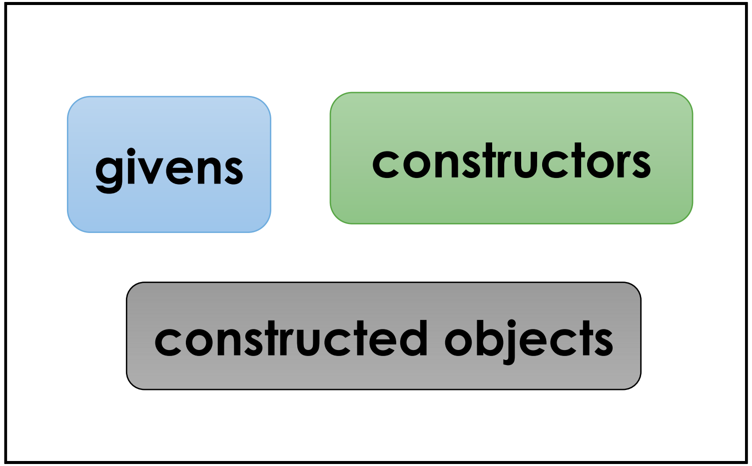
- Reporting on the current state of our work
- First attempt
  - MVP approach
  - We expect to improve and extend our approach in future work.
- Feedback welcome!

Constructional ontology: background

Constructional ontology: the basic idea

1. Start with some objects ("**givens**") or even an empty domain.
2. Construct the rest of the ontology by applying selected **constructors**.

The ontology is characterized by:



Constructional ontology: the basic idea (cont.)

- Generally, the **types** of objects are determined by the constructors that generated the objects.
- The **identity** of constructed objects is dictated by their constructors and the inputs of the constructions.

Gödel on a concept of set

"The concept of set […] according to which a set is anything obtainable from the integers (or some other well-defined objects) by iterated application of the operation 'set of' […]"

(Gödel, *What is Cantor's continuum problem?*)

Recent work

This framework has been recently advocated by Kit Fine.

We build on his ideas.

The Study of Ontology

Kit Fine
U.C.L.A.

A constructional ontology is one which serves to construct complexes from simples. The present paper is concerned with the nature and with the study of such ontologies. It attempts to say, in the first place, how they are constituted and by what principles they are governed. But it also attempts to say how their study may lead one to adopt certain positions and to make certain definitions.

The remarks on the study of ontology are meant to relate to the study of disciplines in general. I am interested in how the study of a discipline gets shaped by the positions which are adopted and the strategies which are pursued. These interact; for the pursuit of certain kinds of strategy will lead to the adoption of certain kinds of position, and the adoption of certain kinds of position will be required by the pursuit of certain kinds of strategy. One therefore needs to understand how they interact.

Certain subsidiary themes run through the paper, all interrelated in one way or another. One concerns a dialectical conception of modality, one that is determined by what is left open at a given stage of enquiry. Another involves a particular way of expressing modal claims, in terms of certain objects requiring others. Yet a third is an interest in a relativist conception of ontology, according to which no ontology stands out as being correct.

The paper concludes with a formal appendix, which attempts to make precise much of what can be made precise in the earlier informal part of the paper. Each part has been designed to be read independently of the other, although a proper understanding of either part depends upon reading them both.

NOÛS 25 (1991) 263-294

THE JOURNAL OF PHILOSOPHY
VOLUME CVII, NO. 11, NOVEMBER 2010

TOWARDS A THEORY OF PART*

My aim in this paper is to outline a general framework for dealing with questions of part-whole. Familiar as this topic may be, my treatment of it is very different from more conventional approaches. For instead of dealing with the single notion of mereological part or sum, I have attempted to provide a comprehensive and unified account of the different ways in which one object can be a part of another. Thus mereology, as it is usually conceived, becomes a small branch of a much larger subject.

My discussion has been intentionally restricted in a number of ways. In the first place, my principal concern has been with the notion of absolute rather than relative part. We may talk of one object being a part of another relative to a time or circumstances (as when we say that the tire was once a part of the car or that the execution of Marie Antoinette was as a matter of contingent fact a part of the French Revolution) or in a way that is not relative to a time or the circumstances (as when we say that this pint of milk is a part of the quart or that the letter 'c' is part of the word 'cat'). Many philosophers have supposed that the two notions are broadly analogous and that what goes for one will tend to go for the other.[1] I believe this view to be mistaken and a source of endless error. But it is not my aim to discuss either the notion of relative part or its connection with the absolute notion.[2]

*The material outlined in this paper has been developed over a period of thirty years. It was most recently presented in a seminar at Princeton in 2000; and I am grateful to Cian Dorr, Michael Fara, Gail Harman, Mark Johnston, David Lewis, and Gideon Rosen for their comments. I am also grateful for some comments I received from Ted Sider and two anonymous referees for this JOURNAL; and I owe a special debt of thanks to Achille Varzi for his encouragement.
[1] As in Ted Sider, *Four Dimensionalism* (New York: Oxford, 2001), for example.
[2] The matter is briefly discussed in Kit Fine, "Things and Their Parts," *Midwest Studies in Philosophy*, XXIII (1999): 61–74.

0022-362X/10/0711/559–589

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Our approach

Core Constructional Ontology (CCO): our current approach

givens

the mereological atoms

We assume that the pluriverse has an atomistic mereological structure.

constructors

set constructor
sum constructor
pair constructor

constructed objects

Key features and benefits of the approach

- Foundational
- Unifying
- Constructional

Foundational

- Object completeness
  - The construction process ("**object factory**") supplies all the objects needed (resulting in an "**object store**").
- Categorical completeness
  - The approach also supplies the **three basic types** of objects (sets, parts, and tuples) together with their associated hierarchical relations.
- Identity criteria
  - The construction determines the conditions for the identity of constructed objects (**extensional** based on the type of constructor and its input).

Unifying

- Common development of three domains (sets, parts, and tuples)
  - Three “domains” (**sets, parts, tuples**) arising in similar ways, i.e. through construction.
- Common basis for identity criteria
  - Identity criteria for objects of the basic types are **extensional**, with differences arising from the way they are constructed.
- Uniform way of capturing key commonalities and differences
  - Commonalities and differences between objects of the basic types can be captured by features of the underlying constructors.

Constructional

- Categorical differences are constructional differences
  - The ways of construction are the basis for differences in kinds of objects.
- Dependency
  - Some objects are built from other objects and hence “depend on” them.
- Reduction
  - The ontology is built out of a relatively small set of fundamental objects.
- Consistency
  - Construction can be a basis for consistency.

Consistency

"The concept of set […] according to which a set is anything obtainable from the integers (or some other well-defined objects) by iterated application of the operation 'set of', […] has never led to any antinomy whatsoever; that is, the perfectly 'naïve' and uncritical working with this concept of set has so far proved completely self-consistent."

(Gödel, *What is Cantor's continuum problem?*)

Core Constructional Ontology and 4-dimensionalism

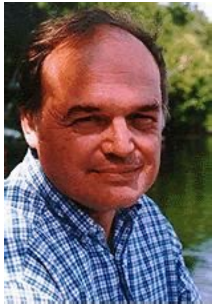
Core Constructional Ontology supplies the required types:

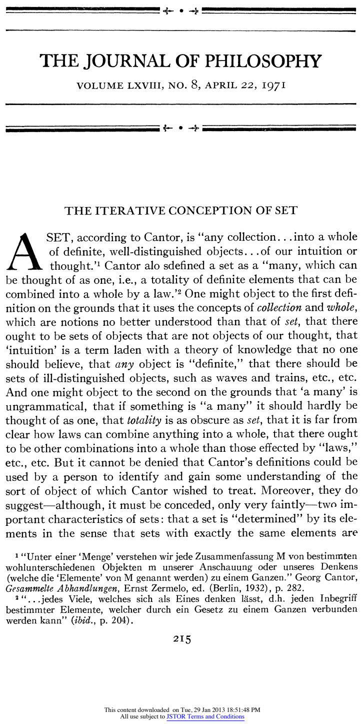
- sets
- sums
- tuples

with the required extensional criteria of identity.

Formalising the Core Constructional Ontology

- A number of options are available.
- For this early phase, we chose a **stage theory**, inspired by
  - Gödel’s remark;
  - George Boolos’s development of the **iterative conception of set** based on a stage theory.





THE JOURNAL OF PHILOSOPHY
VOLUME LXVIII, NO. 8, APRIL 22, 1971

THE ITERATIVE CONCEPTION OF SET

A SET, according to Cantor, is “any collection. . .into a whole of definite, well-distinguished objects. . .of our intuition or thought.’[1] Cantor alo sdefined a set as a “many, which can be thought of as one, i.e., a totality of definite elements that can be combined into a whole by a law.’[2] One might object to the first definition on the grounds that it uses the concepts of *collection* and *whole*, which are notions no better understood than that of *set*, that there ought to be sets of objects that are not objects of our thought, that ‘intuition’ is a term laden with a theory of knowledge that no one should believe, that *any* object is “definite,” that there should be sets of ill-distinguished objects, such as waves and trains, etc., etc. And one might object to the second on the grounds that ‘a many’ is ungrammatical, that if something is “a many” it should hardly be thought of as one, that *totality* is as obscure as *set*, that it is far from clear how laws can combine anything into a whole, that there ought to be other combinations into a whole than those effected by “laws,” etc., etc. But it cannot be denied that Cantor’s definitions could be used by a person to identify and gain some understanding of the sort of object of which Cantor wished to treat. Moreover, they do suggest—although, it must be conceded, only very faintly—two important characteristics of sets: that a set is “determined” by its elements in the sense that sets with exactly the same elements are

[1] “Unter einer ‘Menge’ verstehen wir jede Zusammenfassung M von bestimmten wohlunterschiedenen Objekten m unserer Anschauung oder unseres Denkens (welche die ‘Elemente’ von M genannt werden) zu einem Ganzen.” Georg Cantor, *Gesammelte Abhandlungen*, Ernst Zermelo, ed. (Berlin, 1932), p. 282.

[2] “. . .jedes Viele, welches sich als Eines denken lässt, d.h. jeden Inbegriff bestimmter Elemente, welcher durch ein Gesetz zu einem Ganzen verbunden werden kann” (*ibid.*, p. 204).

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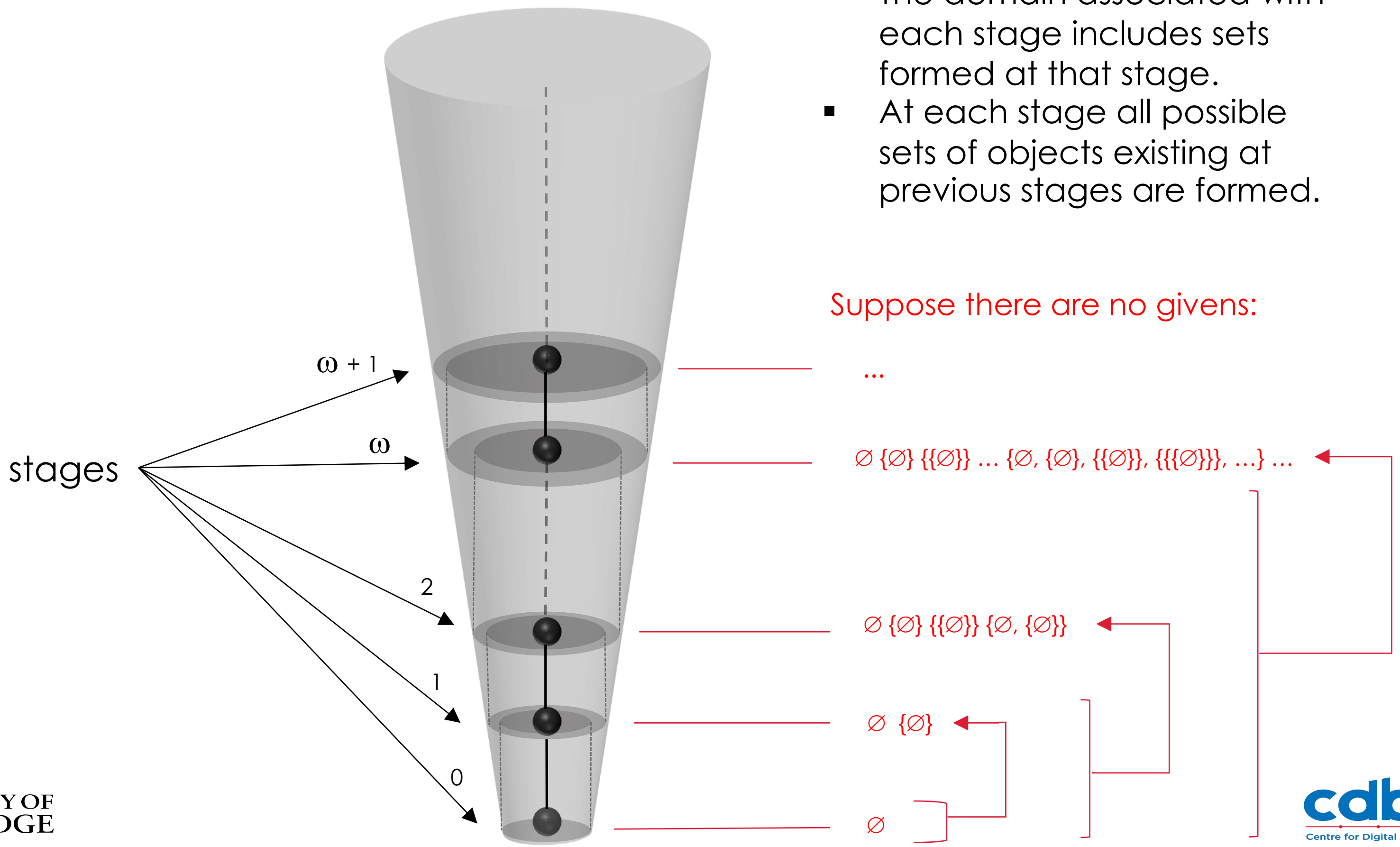
Core Constructional Theory (CCT)

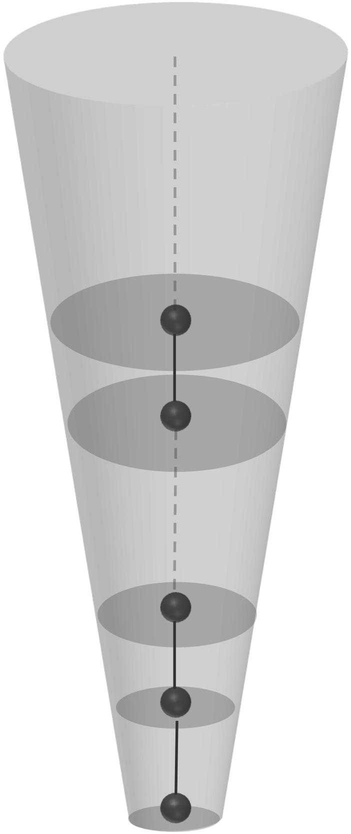
Stage theory (Boolos)

- Stages are well ordered.
- The domain associated with each stage includes sets formed at that stage.
- At each stage all possible sets of objects existing at previous stages are formed.

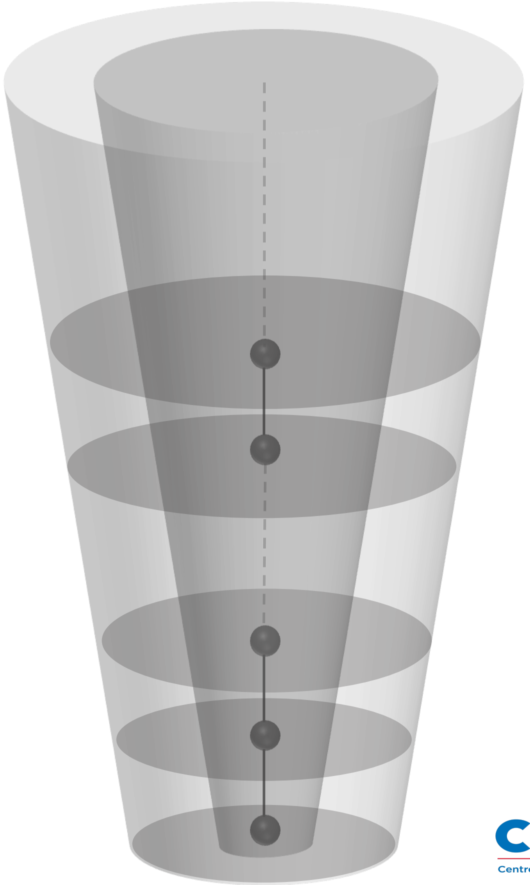
Suppose there are no givens:



Stage theory (Boolos)



starting with
some givens



Core Constructional Theory: novelty

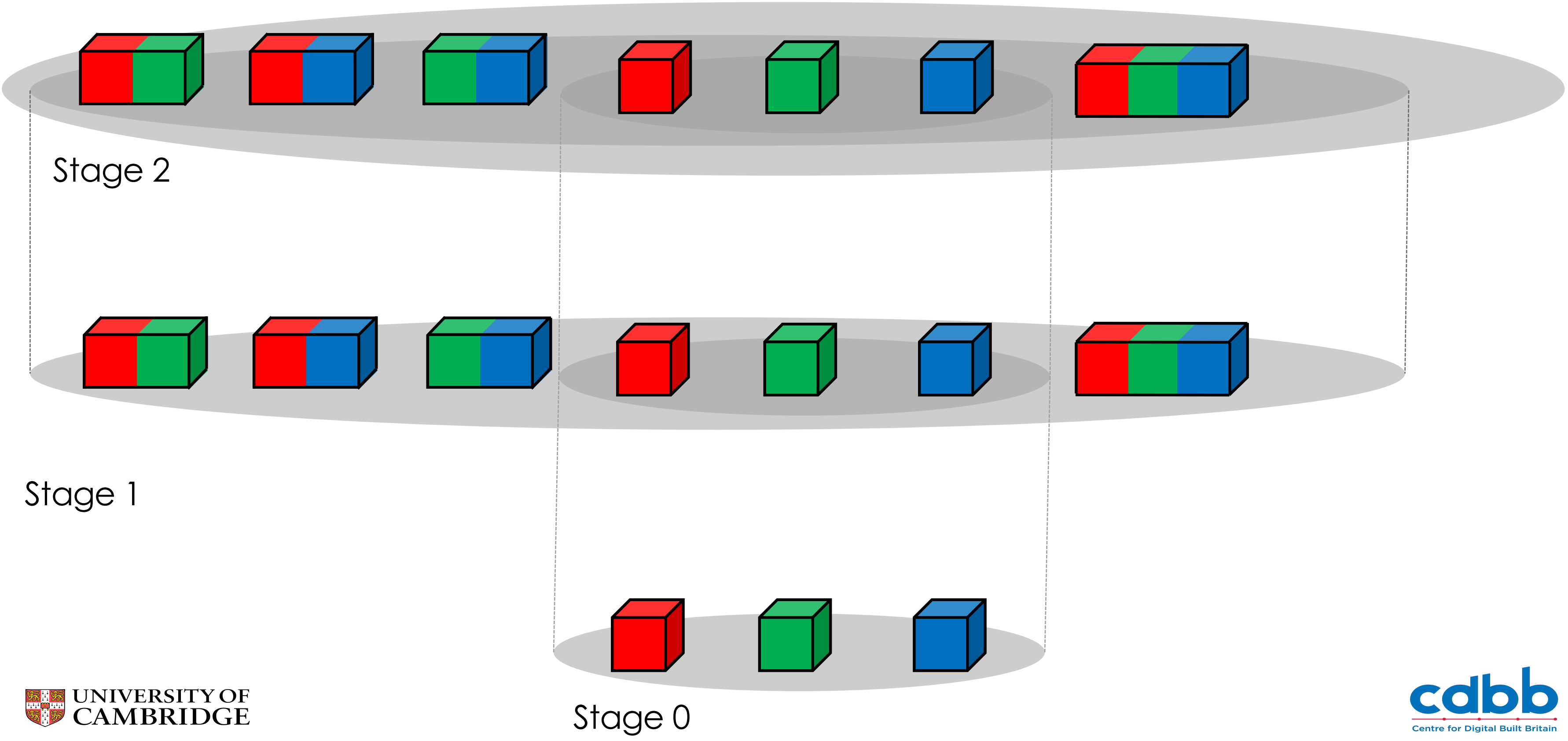
Our work generalizes and extends Boolos's stage theory in three main ways:

1) we provide a unified account of **parts, sets, and tuples**;

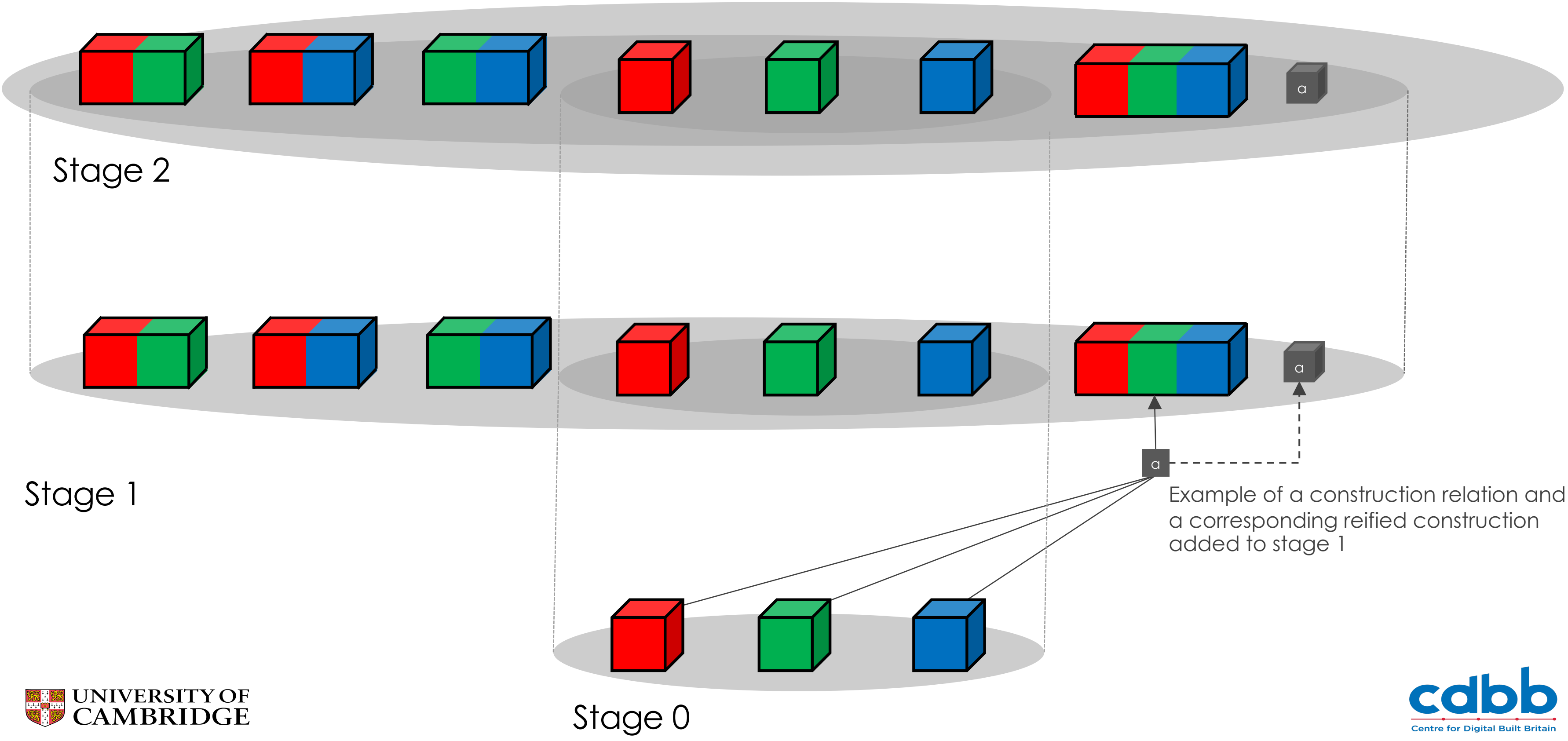
2) we allow a more **flexibile construction process**;

3) in keeping with the target TLOs, CCT includes **reified constructions**, special objects that "log" the structure of the construction process.

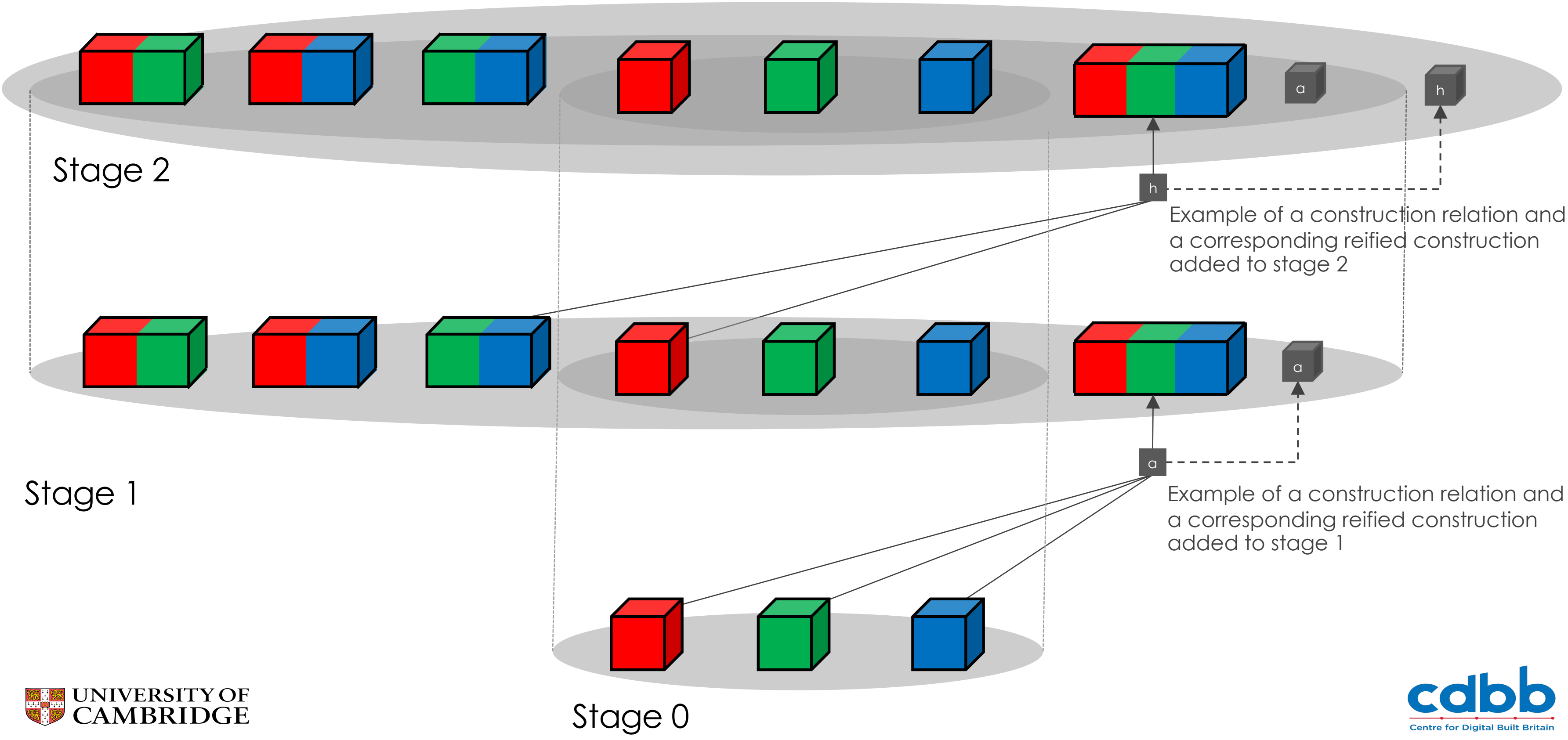
Core Constructional Theory: mereological constructions



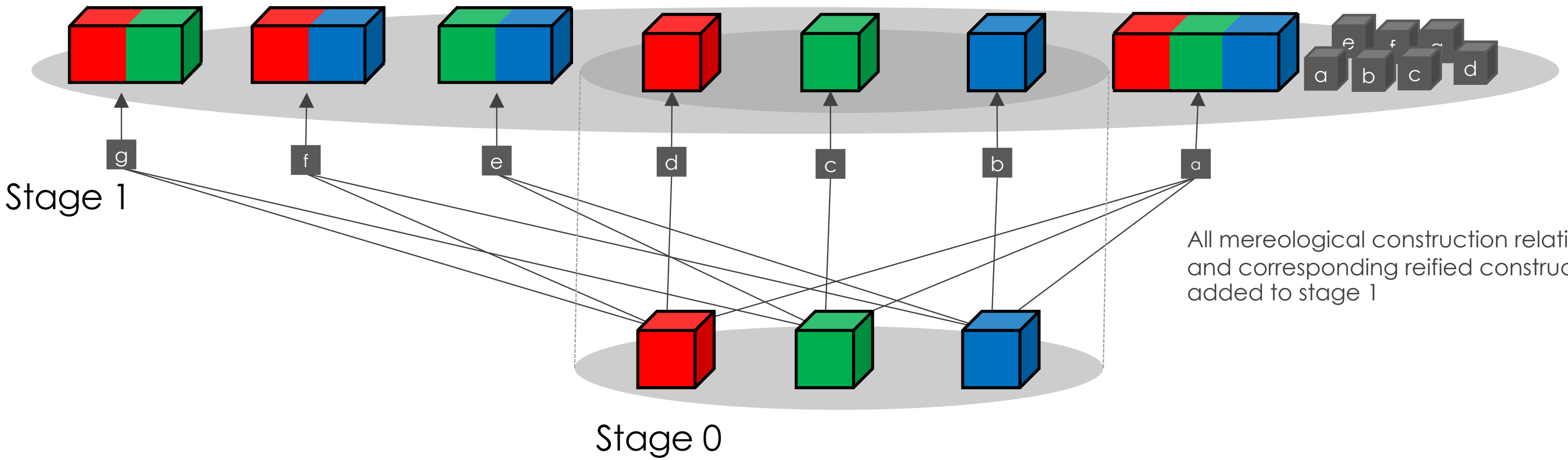
Core Constructional Theory: mereological constructions



Core Constructional Theory: mereological constructions



Core Constructional Theory: mereological constructions



All mereological construction relations and corresponding reified constructions added to stage 1

Logical framework

CCT builds upon a logical framework known as **plural logic**, an extension of standard first-order logic.

This is a **classical two-sorted system**, with singular and plural quantification.

quantification	reading	notation
singular	there is something such that…	$\exists x$
plural	there are some things such that… there is a plurality such that…	$\exists xx$

Logical framework (cont.)

Plural quantification

- gives strength to the theory by allowing to quantify over “collections” of objects in the range of the singular quantifiers.
  - Analogy with classes and monadic second-order quantification
- serves to describe naturally inputs to constructors.

Primitive notions

- Logic
  - Plural logic
- Constructors
  - Set, Sum, Pair
- Construction process
  - Special predicates and constants for types of constructions
- Stages
  - Stage-theoretic notions (is a stage, exists at a stage, follows as a stage)

Axiomatisation

1) Plural logic
2) Stages
3) Initial stages
4) What exists at stages
5) Constructors
6) Reified constructions (“logs” of the construction process)
7) Maximal extension of a stage
8) Classification

Axiomatisation (current draft)

$xx \preccurlyeq yy \leftrightarrow \forall z(z \prec xx \rightarrow z \prec yy)$

$xx \approx yy \leftrightarrow (xx \preccurlyeq yy \wedge yy \preccurlyeq xx)$

$xx@s \leftrightarrow \forall x(x \prec xx \rightarrow x@s)$

$s \lhd t \leftrightarrow s \unlhd t \wedge s \neq t$

$t \unrhd s \leftrightarrow s \unlhd t$

$t \rhd s \leftrightarrow s \lhd t$

$\textsc{Succ}(s,t) \leftrightarrow s \lhd t \wedge \neg\exists u\ (s \lhd u \wedge u \lhd t)$

$\forall xx \exists y\ y \prec xx$

$\forall xx \forall yy[xx \approx yy \rightarrow (\varphi(xx) \leftrightarrow \varphi(yy))]$

$\exists x \varphi(x) \rightarrow \exists xx \forall x(x \prec xx \leftrightarrow \varphi(x))$

$\forall s\ s \unlhd s$

$\forall s \forall t(s \unlhd t \wedge t \unlhd s \rightarrow s = t)$

$\forall s_0 \forall s_1 \forall s_2(s_0 \unlhd s_1 \wedge s_1 \unlhd s_2 \rightarrow s_0 \unlhd s_2)$

$s_0 \unlhd s_1 \wedge s_0 \unlhd s_2 \rightarrow \exists t(s_1 \unlhd t \wedge s_2 \unlhd t)$

$x \lhd y \leftrightarrow x \unlhd y \wedge x \neq y$

$\forall ss \exists s(s \prec ss \wedge \neg\exists t(t \prec ss \wedge t \lhd s))$

$\forall s \exists t\ s \lhd t$

$\exists t(\exists s\ s \lhd t \wedge \forall s(s \lhd t \rightarrow \exists u(s \lhd u \wedge u \lhd t)))$

$xx@s \wedge \forall x(x \prec xx \rightarrow \exists y(\neg\textsc{Stage}(y) \wedge \forall z(\psi(x,z) \leftrightarrow y = z))) \rightarrow \exists t(s \unlhd t \wedge \forall x(x \prec xx \rightarrow \forall y(\psi(x,y) \rightarrow y@t)))$

$\textsc{Init}(s) \leftrightarrow \forall t\ s \unlhd t$

$\textsc{Given}(x) \leftrightarrow \exists s(\textsc{Init}(s) \wedge x@s)$

$\exists x\ \textsc{Given}(x)$

$\textsc{Given}(c_{\text{set}}) \wedge \textsc{Given}(c_{\text{sum}}) \wedge \textsc{Given}(c_{\text{op}}) \wedge \textsc{Given}(c_{\text{union}}) \wedge \textsc{Given}(c_{\text{SetElements}}) \wedge \textsc{Given}(c_{\text{WholeParts}}) \wedge \textsc{Given}(c_{\text{TuplePlaces}}) \wedge \textsc{Given}(c_{\text{SuperSubSets}})$

$\forall x(\neg\textsc{Stage}(x) \rightarrow \exists s\ x@s)$

$s \unlhd t \wedge x@s \rightarrow x@t$

$\text{LUB}(t, ss) \leftrightarrow \forall s(s \prec ss \rightarrow s \unlhd t) \wedge \forall t'(\forall s(s \prec ss \rightarrow s \unlhd t') \rightarrow t \unlhd t')$

$\text{LUB}(t, ss) \rightarrow \forall x(x@t \rightarrow \exists s(s \prec ss \wedge x@s))$

$\textsc{ConstrFrom}(x,s) \leftrightarrow \exists xx(xx@s \wedge \textsc{Set}(x : xx) \vee \textsc{Sum}(x : xx)) \vee \exists u \exists v(u@s \wedge v@s \wedge \textsc{Pair}(x : u, v))$

$\forall x(x@s \leftrightarrow x@t) \rightarrow s = t$

$\textsc{Succ}(s,t) \wedge x@t \rightarrow x@s \vee \textsc{ConstrFrom}(x,s) \vee \textsc{ReifiedConstr}(x)$

$\textsc{Individual}(x) \leftrightarrow (\textsc{Given}(x) \vee \exists xx\ \textsc{Sum}(x : xx))$

$\forall xx \forall s(xx@s \rightarrow \exists t \exists x(s \unlhd t \wedge \textsc{Set}(x : xx) \wedge x@t))$

$\forall x(x@t \wedge \textsc{Set}(x : xx) \rightarrow \exists s(s \lhd t \wedge xx@s))$

$\textsc{Set}(x : xx) \wedge \textsc{Set}(y : yy) \rightarrow (xx \approx yy \leftrightarrow x = y)$

set constructor

$\forall x(x \prec xx \rightarrow \textsc{Individual}(x) \wedge x@s) \rightarrow \exists t \exists x(s \unlhd t \wedge \textsc{Sum}(x : xx) \wedge x@t)$

$\textsc{Sum}(x : xx) \wedge \textsc{Sum}(y : yy) \wedge xx \approx yy \rightarrow x = y$

$\textsc{Sum}(x : xx) \wedge \forall u(u \prec xx \leftrightarrow u = y) \rightarrow x = y$

$\textsc{Sum}(x : xx) \wedge \textsc{Sum}(y : yy) \wedge \exists u\ \exists uu\ \exists vv\ (\textsc{Sum}(u : uu) \wedge \forall z(z \prec xx \leftrightarrow z = u \vee z \prec vv) \wedge \forall z(z \prec yy \leftrightarrow z \prec uu \vee z \prec vv)) \rightarrow x = y$

$x \leq y \leftrightarrow \exists xx\ \exists yy\ (\textsc{Sum}(x : xx) \wedge \textsc{Sum}(y : yy) \wedge xx \preccurlyeq yy)$

$x@s \wedge y@s \rightarrow \exists t \exists z(s \unlhd t \wedge z@t \wedge \textsc{Pair}(z : x, y))$

$\textsc{Pair}(z : x, y) \wedge z@t \rightarrow \exists s(s \lhd t \wedge x@s \wedge y@s)$

$\textsc{Pair}(x : u, v) \wedge \textsc{Pair}(y : u', v') \rightarrow (u = u' \wedge v = v' \leftrightarrow x = y)$

$(\textsc{ConstrProj}_1(w,y) \wedge \textsc{ConstrProj}_1(w,y') \rightarrow y = y') \wedge (\textsc{ConstrProj}_2(w,y) \wedge \textsc{ConstrProj}_2(w,y') \rightarrow y = y') \wedge (\textsc{ConstrProj}_3(w,y) \wedge \textsc{ConstrProj}_3(w,y') \rightarrow y = y') \wedge (\textsc{ConstrProj}_{4a}(w,yy) \wedge \textsc{ConstrProj}_{4a}(w,yy') \rightarrow yy \approx yy') \wedge (\textsc{ConstrProj}_{4b}(w,y_1,y_2) \wedge \textsc{ConstrProj}_{4b}(w,z_1,z_2) \rightarrow y_1 = z_1 \wedge y_2 = z_2)$

$(\textsc{Set}(x : xx) \wedge x@t \wedge \exists s(\textsc{ConstrFrom}(x,s) \wedge s \lhd t)) \rightarrow \exists w(w@t \wedge \textsc{ConstrProj}_1(w, c_{\text{set}}) \wedge \textsc{ConstrProj}_2(w, c_{\text{SetElements}}) \wedge \textsc{ConstrProj}_3(w,x) \wedge \textsc{ConstrProj}_{4a}(w,xx))$

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Axiomatisation (current draft)

The list should be supplemented with axioms stating that ≤ forms an Atomistic General Extensional Mereology (AGEM) whose atoms are precisely the givens.

$$(\textsc{Sum}(x : xx) \land x@t \land \exists s(\textsc{ConstrFrom}(x, s) \land s \lhd t)) \to \exists w(w@t \land \textsc{ConstrProj}_1(w, c_{\text{sum}}) \land \textsc{ConstrProj}_2(w, c_{\text{WholeParts}}) \land \textsc{ConstrProj}_3(w, x) \land \textsc{ConstrProj}_{4a}(w, xx))$$

$$(\textsc{Pair}(x : u, v) \land x@t \land \exists s(\textsc{ConstrFrom}(x, s) \land s \lhd t)) \to \exists w(w@t \land \textsc{ConstrProj}_1(w, c_{\text{op}}) \land \textsc{ConstrProj}_2(w, c_{\text{TuplePlaces}}) \land \textsc{ConstrProj}_3(w, x) \land \textsc{ConstrProj}_{4b}(w, u, v))$$

$$\textsc{Union}(x : yy) \leftrightarrow \exists xx\ (\textsc{Set}(x : xx) \land \forall y(y \prec yy \to \exists zz\ \textsc{Set}(y : zz)) \land \forall z(z \prec xx \leftrightarrow \exists y \exists zz(y \prec yy \land \textsc{Set}(y : zz) \land z \prec zz)))$$

$$(\textsc{Union}(x : yy) \land x@t \land \exists s(yy@s \land s \lhd t)) \to \exists w(w@t \land \textsc{ConstrProj}_1(w, c_{\text{union}}) \land \textsc{ConstrProj}_2(w, c_{\text{SuperSubSets}}) \land \textsc{ConstrProj}_3(w, x) \land \textsc{ConstrProj}_{4a}(w, yy))$$

$$\textsc{ReifiedConstr}(w) \leftrightarrow \exists x\ \textsc{ConstrProj}_1(w, x)$$

$$\textsc{ReifiedConstr}(w) \land w@t \to \exists x \exists xx \exists s(\textsc{Set}(x : xx) \land x@t \land xx@s \land s \lhd t \land \textsc{ConstrProj}_1(w, c_{\text{set}}) \land \textsc{ConstrProj}_2(w, c_{\text{SetElements}}) \land \textsc{ConstrProj}_3(w, x) \land \textsc{ConstrProj}_{4a}(w, xx)) \lor \exists x \exists xx \exists s(\textsc{Sum}(x : xx) \land x@t \land xx@s \land s \lhd t \land \textsc{ConstrProj}_1(w, c_{\text{sum}}) \land \textsc{ConstrProj}_2(w, c_{\text{WholeParts}}) \land \textsc{ConstrProj}_3(w, x) \land \textsc{ConstrProj}_{4a}(w, xx)) \lor \exists x \exists y_1 \exists y_2 \exists s(\textsc{Pair}(x : y_1, y_2) \land x@t \land y_1@s \land y_2@s \land s \lhd t \land \textsc{ConstrProj}_1(w, c_{\text{pair}}) \land \textsc{ConstrProj}_2(w, c_{\text{TuplePlaces}}) \land \textsc{ConstrProj}_3(w, x) \land \textsc{ConstrProj}_{4b}(w, y_1, y_2)) \lor \exists x \exists xx \exists s(\textsc{Union}(x : xx) \land x@t \land xx@s \land s \lhd t \land \textsc{ConstrProj}_1(w, c_{\text{union}}) \land \textsc{ConstrProj}_2(w, c_{\text{SuperSubsets}}) \land \textsc{ConstrProj}_3(w, x) \land \textsc{ConstrProj}_{4a}(w, xx))$$

$$\textsc{Max}(s, t) \leftrightarrow s \unlhd t \land \forall x(\textsc{ConstrFrom}(x, s) \to x@t) \land \forall x(x@t \to \textsc{ConstrFrom}(x, s) \lor (\textsc{ReifiedConstr}(x) \land (\exists y(\textsc{ConstrProj}_3(x, y) \land \textsc{ConstrFrom}(y, s)) \lor (\textsc{ConstrProj}_1(x, c_{\text{union}}) \land \exists yy(yy@s \land \textsc{ConstrProjC}_{4a}(y, yy))))))$$

$$\forall s \exists t\ \textsc{Max}(s, t)$$

$$\textsc{Succ}(s, t) \to \textsc{Max}(s, t)$$

$$\textsc{IsSet}(x) \leftrightarrow \exists xx\ \textsc{Set}(x : xx)$$

$$\textsc{IsPair}(x) \leftrightarrow \exists y_1 \exists y_2\ \textsc{Pair}(x : y_1, y_2)$$

$$\begin{aligned}
&(\textsc{IsSet}(x) \to \neg\textsc{Individual}(x) \land \neg\textsc{IsPair}(x) \land \neg\textsc{ReifiedConstr}(x) \land \neg\textsc{Stage}(x)) \land \\
&(\textsc{Individual}(x) \to \neg\textsc{IsSet}(x) \land \neg\textsc{IsPair}(x) \land \neg\textsc{ReifiedConstr}(x) \land \neg\textsc{Stage}(x)) \land \\
&(\textsc{IsPair}(x) \to \neg\textsc{IsSet}(x) \land \neg\textsc{Individual}(x) \land \neg\textsc{ReifiedConstr}(x) \land \neg\textsc{Stage}(x)) \land \\
&(\textsc{ReifiedConstr}(x) \to \neg\textsc{IsSet}(x) \land \neg\textsc{Individual}(x) \land \neg\textsc{IsPair}(x) \land \neg\textsc{Stage}(x)) \land \\
&(\textsc{Stage}(x) \to \neg\textsc{IsSet}(x) \land \neg\textsc{Individual}(x) \land \neg\textsc{IsPair}(x) \land \neg\textsc{ReifiedConstr}(x))
\end{aligned}$$

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Axiomatisation: set constructor

Key axioms for the **set constructor**

(15) $\forall xx \forall s (xx@s \;\rightarrow\; \exists t \exists x (s \trianglelefteq t \wedge \boxed{\textsc{Set}(x : xx)} \wedge x@t))$

(For every plurality xx of objects existing at s, there is a later stage t at which the set of xx exists.)

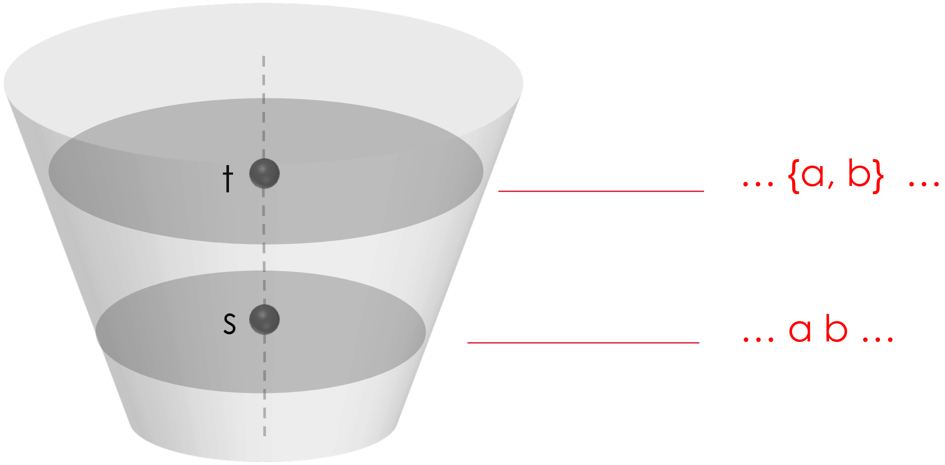
(16) $\forall x (x@t \wedge \boxed{\textsc{Set}(x : xx)} \rightarrow \exists s (s \triangleleft t \wedge xx@s))$

(The elements of a set exist at an earlier stage than the set itself.)

(17) $\boxed{\textsc{Set}(x : xx)} \wedge \boxed{\textsc{Set}(y : yy)} \rightarrow (xx \approx yy \leftrightarrow x = y)$

(Extensionality: two sets are identical if and only if their elements are the same.)

Axiomatisation: set constructor (cont.)



- Suppose the plurality of a and b exists at stage s and not before s.
- Then the set of a and b, {a, b}, exists at a stage t after s.

Set theory and mereology in CCT

Using a natural definition of membership, we can **deduce** the axioms of **Zermelo-Fraenkel (ZF) set theory**, minus Empty Set, in CCT.

Currently, the axioms of **Atomistic General Extensional Mereology (AGEM)** are incorporated after defining parthood. At the next stage, we will drop the axioms and deduce them from CCT.

Consistency

We provide a **mathematical proof of consistency** of CCT.

This is done by constructing a **model** within some chosen **metatheory**.

CCT is then shown to be consistent relative to this metatheory.

- Morse-Kelley class theory (MK): it adds to ZFC a single layer of classes on top of the sets
- ZFC + an extra axiom stating that there exists an inaccessible cardinal
- ZFC for weakenings of CCT (e.g. plural comprehension restricted to stages or dropping the analogue of Replacement)

CLIF

- To help ensure logical data quality, Paweł Garbacz is working on translating automatically the human-readable axioms of CCT into CLIF.
- This will avoid manual translation errors.
- Owing to the axiom schemas in CCT, the translation is lossy.
- We anticipate further lossy translations to OWL.

Questions and feedback

Core Constructional Ontology (CCO): a Constructional Theory of Parts, Sets, and Relations

4-Dimensionalism in Large Scale Data Sharing and Integration

Newton Gateway to Mathematics

Appendix: informal overview of the axiomatisation

1) Plural logic
   - Pluralities are non-empty.
   - Pluralities with the same members satisfy the same formulas.
   - There is a plurality corresponding to every formula satisfied by one thing ("If there is an F , then there are the Fs.").
2) Stages
   - Stages form a convergent, serial partial order.
   - Stages are well founded.
   - There are infinitely many stages and a limit stage.
   - A version of the axiom of Replacement holds for stages

Appendix: informal overview of the axiomatisation

3) Initial stages

- The initial stage is non-empty (there are “givens” at this stage).
- We assume the existence of specific givens serving to represent constructors and other relevant relations (set-elements, whole-parts, tuple-places, super-subsets).

Appendix: informal overview of the axiomatisation

4) What exists at stages

- Everything that isn't a stage exists at some stage.
- Stages are "cumulative" (everything that exists at earlier stages also exists at later stages).
- Limit stages are "collection" stages.
- Stages with identical domains are identical.
- What exists at a successor stages existed before or resulted from some construction.

Appendix: informal overview of the axiomatisation

5) Constructors

5.1) Set constructor

- Every plurality of objects at a stage is used to construct a set.
- The elements of a set exist before the set.
- Extensionality (two sets are identical iff they have the same elements)

Appendix: informal overview of the axiomatisation

5.2) Sum constructor

- Every plurality of individuals at a stage is used to construct a sum.
- Sums constructed from the same pluralities are the same.
- The sum constructed from the singleton plurality of x is x.
- Two pluralities, one obtained from the other by replacing some objects with their sum, yield the same sum.
- Parthood satisfies the axioms of AGEM.

Appendix: informal overview of the axiomatisation

5.3) Pair constructor

- For any two objects existing at a stage, there is a later stage when they are used to construct a pair.
- The coordinates of a pair exist before the pair.
- Extensionality (two pairs are identical iff their first coordinates are the same and their second coordinates are the same)

Appendix: informal overview of the axiomatisation

6) Reified constructions ("logs" of construction process)

- These axioms ensure that, whenever certain constructions are effected, there are objects that encode this information.

7) Maximal extension of a stage

- These axioms sanction that every stage *s* has a maximal extension, i.e. a stage obtained by effecting every construction possible at *s*.

8) Classification

- These axioms partition the domain of the theory in five kinds of entities: individuals, sets, pairs, reified constructions, and stages.